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String Cosmology in Brans–Dicke Theory for Kasner Type Metric

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Abstract For a string Bianchi type-I metric of Kasner form in Brans–Dicke theory of gravity, it is not possible to describe an anisotropic physical model of the universe.

Keywords Brans–Dicke theory of gravity · Cosmic strings · Kasner models

1 Introduction

We have no definite idea about the kinds of matter fields present in the early universe. Though the universe, on a large scale, seems homogeneous and isotropic at the present time, there are no obervational values which predict the isotropy in an era prior to the recombination. In fact, it is possible to begin with an anisotropic universe which isotropizes during its evolution by the damping of this anisotropy via a mechanism of viscous dissipation. These anisotropies have many possible sources and could be associated with cosmological magnetic or electric fields, long-wavelenth gravitational waves, Yang–Mills fields, axion fields in low energy string theory or topological defects such as cosmic strings or domain walls etc.

In Einstein's theory (with cosmological constant $\Lambda = 0$) the Kasner [1] universe refers to a vacuum cosmological model. The generalizations of Kasner model were proposed by Henekman and Schucking [2], Misner [3], Lifshitz–Khalatnikov group [4], Belinski [5, 6]. Gron [7, 8] has defined an analytic nondimensional expression for the anisotropy of the Kasner metric. Barrow [9], Caltaldo [10], Brevik and Petterson [11, 12] proved that a viscous cosmological fluid does not permit the Kasner metric to be anisotropic in Einstein's general relativity.

Now a days, Brans–Dicke [13] theory of gravity is more important amongst all existing alternative theories of gravitation. The latest inflationary model [14], extended inflation [15, 16], hyper extended inflation and extended Chaotic inflation [17] are based on Brans–Dicke theory and general scalar-tensor theories.

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Brans–Dicke [13] formulated a scalar-tensor theory of gravitation in which gravity is mediated by a scalar field ϕ in addition to the usual metric tensor field g_{ij} present in Einstein theory. In this theory, the long range scalar field ϕ is generated by the whole of matter in the universe according to Mach's Principle [18] and has the dimension of the universe of the gravitational constant G.

The field equations in Brans–Dicke theory are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{'k}\right) - \phi^{-1}(\phi_{i;j} - g_{ij}\Box\phi) \tag{1}$$

and

$$\Box\phi = \phi^{'k}_{;k} = 8\pi\phi^{-1}T(3+2\omega)^{-1}, \tag{2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, T_{ij} is the stress energy of the matter, and comma and semicolon denote partial and covariant differentiation respectively. The equations of motion $T^{ij}_{;j} = 0$ are consequences of the field equations (1) and (2).

The concept of string theory was developed to describe events at the early stages of the evolution of the universe. Kibble [19] and Vilenkin [20] believed that strings may be one of the sources of density perturbations that are required for the formation of large scale structures in the universe. The study of string cosmological models was initiated by Vilenkin [21], Letelier [22], Krori et al. [23, 24]. Relativistic string models in the context of Bianchi-space time have been obtained by Krori et al. [23], Banerjee et al. [25], Tikekar [26], Bhattacherjee and Baruah [27], Mahanta and Mukherjee [28]. Gundlach and Ortiz [29], Barros and Romero [30], Sen and Banerjee [31], Barros et al. [32], Reddy [33, 34] have studied several aspects of cosmic strings in Brans–Dicke [13] and Saez–Ballester [35] scalar-tensor theories of gravitation.

In this paper we consider an anisotropic Bianchi type-I model of Kasner form in Brans–Dicke scalar-tensor theory of gravitation in the presence of cosmic string source.

2 Metric and Field Equations

We consider an anisotropic Bianchi type-I metric of Kasner form in Brans–Dicke scalar-tensor theory of gravitation in the presence of cosmic string source as

$$ds^2 = dt^2 - t^{2p_1}dx^2 - t^{2p_2}dy^2 - t^{2p_3}dz^2, \tag{3}$$

where p_1, p_2 and p_3 are three parameters that we shall require to be constants.

The energy momentum tensor for cosmic string source is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j, \tag{4}$$

where ρ is the rest energy density of cloud of strings with particles attached to them, λ is the tension density of strings, u^i the cloud four velocity and x^i is the direction of anisotropy.

We have

$$u^i u_i = -x^i x_i = 1 \quad \text{and} \quad u^i x_i = 0. \tag{5}$$

We consider $\rho = \rho_p + \lambda$, where ρ_p is the rest energy density of particles and x^i to be along x-axis, so that

$$x^i = (t^{-p_1}, 0, 0, 0) \quad \text{and} \quad u^i u_i = (0, 0, 0, 1). \tag{6}$$

We first find the components of the Ricci tensor R_{ij}. Assuming the metric, the nonvanishing components of the Christoffel symbols are

$$\Gamma^4_{ii} = p_i t^{2p_i - 1}, \qquad \Gamma^i_{i4} = p_i/t, \quad i = 1, 2, 3.$$

So we calculate

$$R_{ii} = p_i(p_1 + p_2 + p_3 - 1)t^{2p_i - 2},$$
$$R_{44} = [(p_1 + p_2 + p_3) - (p_1^2 + p_2^2 + p_3^2)]t^{-2}.$$

Let $S = p_1 + p_2 + p_3$, $\theta = p_1^2 + p_2^2 + p_3^2$ and $R = [S^2 - 2S + \theta]t^{-2}$.
Using (4), (5) and (6), the components of (1) can be written as

$$G^1_1 = -8\pi\phi^{-1}\lambda - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 - \frac{\phi_4}{\phi}\left(\frac{p_2}{t} + \frac{p_3}{t}\right) - \frac{\phi_{44}}{\phi},$$

$$G^2_2 = \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_1}{t} + \frac{p_3}{t}\right) + \frac{\phi_{44}}{\phi},$$

$$G^3_3 = \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_1}{t} + \frac{p_2}{t}\right) + \frac{\phi_{44}}{\phi},$$

$$G^4_4 = -8\pi\phi^{-1}\rho - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_1}{t} + \frac{p_2}{t} + \frac{p_3}{t}\right).$$

The field equations (1), (2) for the metric (3) with the help of (4), (5) and (6) can be written as

$$\left[p_1(S-1) - \frac{1}{2}(S^2 - 2S + \theta)\right]t^{-2} = -8\pi\phi^{-1}\lambda + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_2}{t} + \frac{p_3}{t}\right) + \frac{\phi_{44}}{\phi},$$

$$\left[p_2(S-1) - \frac{1}{2}(S^2 - 2S + \theta)\right]t^{-2} = \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_1}{t} + \frac{p_2}{t}\right) + \frac{\phi_{44}}{\phi}, \tag{7}$$

$$\left[p_3(S-1) - \frac{1}{2}(S^2 - 2S + \theta)\right]t^{-2} = \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_1}{t} + \frac{p_2}{t}\right) + \frac{\phi_{44}}{\phi},$$

$$\frac{1}{2}(\theta - S^2)t^{-2} = -8\pi\phi^{-1}\rho - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + \frac{\phi_4}{\phi}\left(\frac{p_1}{t} + \frac{p_2}{t} + \frac{p_3}{t}\right), \tag{8}$$

$$\left(\frac{p_1}{t} + \frac{p_2}{t} + \frac{p_3}{t}\right)\phi_4 + \phi_{44} = \frac{8\pi\,\phi^{-1}}{(3 + 2\omega)}(\rho + \lambda), \tag{9}$$

$$\rho_4 + \rho\left(\frac{p_1}{t} + \frac{p_2}{t} + \frac{p_3}{t}\right) - \lambda\frac{p_1}{t} = 0, \tag{10}$$

where the suffix 4 following an unknown function denotes ordinary differentiation with respect to time.

From the set of equations (7) one can obtain the equation.

$$\left[-\frac{1}{2}S^2 + 2S - \frac{3}{2}\theta\right]t^{-2} = -8\pi\phi^{-1}\lambda + \frac{3}{2}\omega\left(\frac{\phi_4}{\phi}\right)^2 + 2\left(\frac{p_1 + p_2 + p_3}{t}\right)\frac{\phi_4}{\phi} + 3\frac{\phi_{44}}{\phi}. \tag{11}$$

Using (8) and (11), ρ and λ can be explicitly expressed as

$$8\pi\rho = \frac{(S^2-\theta)\phi}{2t^2} - \frac{\omega}{2}\frac{\phi_4^2}{\phi} + \frac{S}{t}\phi_4, \tag{12}$$

$$8\pi\lambda = \frac{(S^2-4S+3\theta)\phi}{2t^2} + \frac{3}{2}\omega\frac{\phi_4^2}{\phi} + 2\frac{S}{t}\phi_4 + 3\phi_{44}. \tag{13}$$

Substituting values of ρ and λ from (12) and (13) into (9), we get

$$\phi_{44} + \frac{S}{t}\phi_4 = \frac{1}{(3+2\omega)}\left[\frac{(S^2-2S+\theta)}{t^2} + \omega\left(\frac{\phi_4}{\phi}\right)^2 + 3\frac{S}{t}\frac{\phi_4}{\phi} + 3\frac{\phi_{44}}{\phi}\right] \tag{14}$$

which is similar to result obtained by Mauricio Cataldo et al. [10]. We should note that, when $\phi = \text{constant} = G^{-1}$ (14) coincides with the results obtained by Brevik and Petterson [12].

The above set of field equations are highly non-linear and hence to obtain its determinate solution, we assume

$$\rho + \lambda = 0, \quad \text{(Ready [33, 34])}.$$

Adding (12) and (13), we get

$$8\pi(\lambda+\rho) = \frac{(S^2-2S+\theta)}{t^2} + \omega\frac{\phi_4^2}{\phi^2} + 3\left(\phi_{44} + \frac{S}{t}\phi_4\right). \tag{15}$$

Using (9) and (15), it simplifies to

$$\left(S^2-2S+\theta\right)t^{-2} + \omega\frac{\phi_4^2}{\phi^2} = 0. \tag{16}$$

3 Conclusion

On examining equation (16), we conclude that "For a string Bianchi type-I metric of Kasner form in Brans–Dicke scalar tensor theory of gravitation, it is not possible to describe an anisotropic physical model of the universe". Which is again similar to the result obtained by Cataldo et al. [10] identical to what is found in Einstein's theory.

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